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Firm-level Resource Allocation to Information Security in the Presence of Financial Distress

By

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Firm-level Resource Allocation to Information Security in the Presence of Financial Distress

Abstract

In this paper, we adopt an organizational perspective to the management of information security and analyze in a multi-period context how an organization should allocate its internal cash flows and available external funds to revenue-generating (productive) and security assuring (protective) processes in the presence of security breach, borrowing and financial distress costs. We show analytically and illustrate numerically that the capital stock accumulation is lower and allocations to security are higher in the initial periods when security breach costs are higher. We also show that the steady state capital accumulation is not different in the presence of security breach and financial distress costs compared to the no-breach case. Further, we show that external insurance can be beneficial to both the firm and the provider and examine the cost parameters that affect the feasibility range. The results highlight the importance of resource allocation and insurance at the organizational level in addressing security breach problems and enable managers to seek and use relevant information effectively.

Keywords: Security Breach Costs; Financial Distress; Insurance; Resource Allocation.

1. INTRODUCTION

Organizations have become more dependent on the use of digital technologies for operational, investment, financing and strategic decisions. While this has resulted in greater efficiency and effectiveness in organizational decision making at all levels, it has also made organizations more vulnerable to information security breaches in two respects. On the one hand, it has increased the scope and value of potential benefits to the perpetrators of security attacks encouraging them to attempt more attacks. Further, because the decisions in an organization are inter-related, a breach in any part of the organization can proliferate into other parts, making successful attacks in one part cause financial distress to the entire organization. It is widely reported that the frequency of security breaches is rapidly increasing and their costs are doubling each year (Garg et al. 2003; Bagchi and Udo 2003; Lukasik 2000). A case in point is TJX, the fashion retailer which has released details of a recent security breach that has cost the firm an estimated $17 million to date, excluding the losses from exposure to potential legal proceedings and other liabilities and costs (Murphy, 2007).

If these information security breach costs are not adequately controlled, they might exceed the combined internal and external funds available to a firm. If this were to occur, the firm would face financial distress. Adequate and judicious allocation of resources to preventive and corrective security measures would reduce the chance of financial distress. Some examples of such actions include improvements in security technologies, security-focused managerial practices, effective governance structures and purchase of external insurance.

In this paper, we adopt an organizational perspective to the management of information security. More specifically, we analyze how an organization could allocate its internal cash flows and available external funds to revenue-generating (productive) and security assuring (protective) processes. The difficulty of effective resource allocation under circumstances characterized by the uncertain nature and severity of breach costs has been pointed out by Rue et al. (2007). In contrast to prior literature, we examine this allocation process by explicitly considering the possibility that a firm could face financial distress with accompanying additional costs related to reorganization and recovery. However, we do not assume that financial distress necessarily leads to liquidation of the firm. In reality, firms continue to operate under financial distress, albeit with additional costs. Our model captures this reality. In addition, a firm can purchase external insurance to mitigate security breach costs. For example, the CSI/FBI 2006 survey reports that 29 percent of the respondent firms have purchased cyber-insurance which is an increase of 4 percent from the previous year (Gordon et al. 2006). This suggests increasing use of external insurance by firms. We study how organizations could benefit from the purchase of such insurance.

The cost of security-assuring operations include both self-protection (prevent and/or deflect security attacks) and information recovery costs. We develop a multi-period model in which the firm allocates resources to revenue generating and security assuring operations at the beginning of each period. In this model, financial distress is incorporated as an aggregate cost that includes both direct costs such as legal and administrative costs of reorganization and indirect costs such as impaired ability to conduct business and agency costs. [1] We then examine potential benefits of purchasing insurance from an external provider. In our model, insurance is exogenous and is purchased by an individual *firm* to protect its information assets rather than by individual *users* who may purchase insurance to protect against their losses arising from the use of firm's products. The need for external insurance derives either from inherent risk aversion of the firm or from financial distress. Out of these two drivers,

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[1] For a full description of financial distress costs, refer to Ross et al. (2008)

our focus is primarily on financial distress. Our findings are therefore applicable to risk-neutral firms.

The models developed in this paper and the findings regarding external insurance not only enhance our understanding of allocation of resources to security at the organizational level but also have implications for accounting. Our model requires a systematic collection of security breach costs and an understanding of firm's constraints in raising incremental capital to counter excessive security breach costs. The organization can implement the model by developing an internal data collection and classification infrastructure to track the costs of security breach and an organizational mechanism to integrate this data with borrowing and other financing constraints typically known to the corporate finance department. Further, the market for insurance requires assessment of risks by insurance companies which in turn requires widespread information availability on the pattern of security breach costs in different industries, locations and contexts. Financial accounting systems that require reporting of these costs not only help in their collection but also reduce information asymmetry between insurance companies and firms that need the insurance.

To the best of our knowledge, this is the first study to examine at the broad organizational level how resources should be allocated to revenue generating and security assuring processes when faced by potential financial distress. The remainder of the paper is organized as follows. The next section discusses prior related research. Section 3 gives the overview, notation and an initial model without any security breaches. Section 4 provides the detailed model, results and numerical illustrations for the scenario with security breach, borrowing and financial distress costs. The role of external insurance is examined in Section 5. Summary and Concluding remarks are provided in Section 6.

2. RELATED WORK

Analysis of information security allocation from an economic perspective has recently attracted the interest of many researchers. For example, Gordon and Loeb (2002) present an economic model that determines optimal allocation among different information assets with different vulnerabilities but do not explicitly consider the broader firm level resource allocation between revenue generating and security assuring activities. Kumar et al. (2007) explore firm level security budgeting when decision rights reside with different agents who might have divergent priorities. Researchers such as Cavusoglu et al. (2005) have examined the economic value of individual security technologies such as Intrusion Detection Systems that can be deployed by a firm. This research stream, however, has focused on the relative importance of different types of security-related expenditures but not on the broader trade-off involved between revenue generating and security assuring operations. This trade-off is indeed the first stage of decision making at the firm level before any detailed budgeting for different security technologies and contexts is undertaken. Further this trade-off could be affected by the possibility of financial distress and its accompanying costs. We complement current literature by modeling the above trade-off in the presence of borrowing and financial distress costs in a multi-period setting.

Apart from making investments in security assuring operations, a firm could also consider purchasing insurance from an external provider to mitigate its exposure to financial distress resulting from security breaches. A number of researchers have examined the feasibility of such external insurance. Kesan et al. (2005) justify the purchase of external insurance and point out that insurance encourages adoption of socially optimal security standards and could prevent a market failure when risks are not transferable. Recently, Bolot and LeLarge (2008) have developed an analytical model in which they examine the role of external insurance in mitigating correlated security risks for individual users on the Internet. At the firm level, Gordon et al. (2003) discuss the viability of external insurance as a way to hedge against information security breach costs. Ogut et al. (2005) develop a model of information security investment when the firms are assumed to be risk averse and their security risks are interdependent. In this paper, we do not base our analysis on firms' risk aversion or mutual interdependence. Instead, the drivers for external insurance in our paper are financial distress and borrowing costs even when the firms and the insurance providers are risk-neutral.

3. OVERVIEW, NOTATION AND THE NO-BREACH MODEL

We formulate a finite multi-period model for the allocation of resources to revenue generating and security assuring operations at the beginning of each period $t = 0, 1, \ldots T$. The firm has an initial productive capital endowment K_0. The revenue generating processes produce cash inflows for the firm. On the other hand, the security assuring operations enhance technological, managerial and organizational measures that reduce the cost of security breaches. Security breaches impose a variety of costs on the firm, such as loss of credibility, loss of asset value and the loss of time and effort required for recovery. In effect, these costs deplete the residual cash (funds available after investments in productive and security operations) available to the firm. One consequence of this depletion is that a firm will have less resources to invest in revenue generating processes and to pay dividends in future time periods. It is possible that security assuring processes provide competitive advantage to the firm and thereby increase revenues. Our model incorporates this possibility.[2]

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[2] If the marginal revenue from allocation to security allocation is higher than the marginal revenue from allocation to revenue generating activities, there is no issue of allocation as such. If the marginal revenue from allocation to security is less than the marginal revenue from allocation to revenue

If the breach costs are significantly large, the internal funds might not be sufficient to cover those costs and therefore, the firm will be forced to borrow [3] from the capital market and incur borrowing costs. In some cases, the breach costs could be so large that the firm might face additional constraints in borrowing the required amount. We define this condition as financial distress while assuming that the firm would continue to operate. Thus the firm is faced with the problem of how to manage current and future information security breach costs through investments in security assuring and revenue generating activities. The allocation decisions made in earlier periods affect future investments, breach costs and residual cash available to the firm. The nature of this decision making suggests the use of a dynamic model to generate the optimal allocation of funds in each period.

In the first phase, we develop an initial scenario without security breach. This scenario is the first-best case. We introduce the possibility of security breach and financial distress in the second phase (section 4). Comparing that scenario with the no-breach case, we can show how security breach along with financial distress deviate the firm's decisions from the first-best case. Finally, in the third phase (Section 5), we show how external insurance can be helpful by allowing the firm to purchase external insurance that covers part of the information security breach costs and thereby reduces the probability of financial distress.

In the Table 1, we present the basic notation. Next, we develop the framework that underlies all the models.

Insert Table 1 here

generating activities, the opportunity cost of security allocation is the difference between the two. The cost of security allocation is that opportunity cost.

[3] If the original capital structure includes debt, the firm will be forced to increase its borrowing to meet the cost of security breaches that is not covered by internal funds.

Revenue Generation

The firm generates net cash revenue based on the production function

$$y_{t+1} = F(K_{t+1}) \qquad (1)$$

$$K_{t+1} = (1-\beta)K_t + k_t \qquad t = 0,1,..T \qquad (2)$$

In the above expression K_t is the capital stock at the beginning of period t and k_t is the new investment in revenue generating operations in period t. The initial capital stock K_0 is given[4]. β is a factor that specifies the proportion of the capital stock that is used for replacement and maintenance of capital assets in any time period; $F(\cdot)$ is assumed to be twice differentiable and satisfies the following conditions: $F'(\cdot) > 0, F''(\cdot) < 0, F(0) = 0, F'(0) = +\infty, F(+\infty) = +\infty$, and $F'(+\infty) = 0$. Note that the net cash revenue as defined here is net of all cash expenditures including any interest payment on the initial debt and excludes accruals such as credit sales and other non-cash revenues and expenses.

Security Assurance

The information security breach cost that we consider is not asset-specific and therefore, individual asset vulnerabilities are not explicitly modeled. The firm-level aggregate security breach cost is assumed to be a random variable $\xi_t \in [\xi_m, +\infty)$ which is distributed as a Pareto distribution with the density function $\rho(\xi_t) = \frac{\delta \cdot \xi_m^{\delta}}{\xi_t^{\delta+1}}$, where $\xi_m > 0$ is the location parameter and $\delta > 2$ (to ensure the existence of finite mean and variance) is the shape parameter. Therefore, the probability that the breach cost is greater than $\xi \in [\xi_m, +\infty)$ is given by

[4] For the purposes of our analysis, we assume that the firm gives the same expected risk-adjusted returns to investors as they can get by reinvesting dividends.

$$\Pr(\xi_t > \xi) = \left(\frac{\xi}{\xi_m}\right)^{-\delta} \text{ for all } \xi \geq \xi_m \quad (3)$$

The motivation for using Pareto distribution is to capture the intuition of Power Law which implies that most breaches cause only a small financial loss, whereas a few breaches may cause very large financial loss to the firm (long tail). This distribution is widely used for insurance modeling (Hausken 2006; Rust and Phelan 1997).

The expected cost resulting from information security breach is

$$E\{\xi\} = \frac{\delta \cdot \xi_m}{\delta - 1} \quad (4)$$

In each period, the firm can reduce the breach cost by investing in security assuring processes. We model the location parameter of the Pareto distribution as a strictly decreasing convex function of security allocation s_t such that

$$\Pr(\xi_t > \xi) = \left(\frac{\xi}{\xi_{mt}}\right)^{-\delta} \text{ for all } \xi \geq \xi_{mt} \text{ and } \xi_{mt} = h(s_t). \quad (5)$$

The function $h(s_t)$ is characterized by $h'(s_t) < 0, h''(s_t) > 0$, $h(0) = \bar{h}$, $h(+\infty) = \underline{h}$, and $\bar{h} > \underline{h} > 0$. This implies that the expected value $E(\xi_t)$ is twice differentiable and is decreasing in s_t. **The Firm's Decision Problem**

The firm's objective is to maximize the present value of future net cash flows over the planning horizon plus the wealth at the end of the planning horizon. We abstract away from accruals and accounting profits in our value maximization model. Our analysis proceeds as follows. We first model a scenario with no security breaches and derive the capital stock accumulation over time

Model with no security breaches In the absence of breach costs, the firm need not invest in security assuring processes. Therefore, the one-period residual cash flow available to the firm in period t is $d(K_t, K_{t+1}) = y_t - k_t$. Substituting for k_t from (2), we have $d(K_t, K_{t+1}) = y_t - k_t = F(K_t) - K_{t+1} + (1-\beta)K_t$. The wealth of the firm at period *t* is $W(K_t) \equiv F(K_t) + (1-\beta)K_t$ ..

Given an initial capital stock of K_0, the firm's problem is to decide capital accumulations $\{K_1, K_2, ..., K_{T+1}\}$ [5] to maximize the present value of net cash flows plus the wealth of the firm at the terminal date $T+1$, that is,

$$V_0(K_0) = \underset{\{K_{t+1}\}_{t=0}^{T}}{Max} \sum_{t=0}^{T} \phi^t d(K_t, K_{t+1}) + \phi^{T+1} W(K_{T+1}) \tag{6}$$

where $\phi \in (0,1)$ is the discount factor; $V_0(K_0)$ is the value of the firm at the initial period. The dynamic optimization problem in (6) can be expressed as the following recursive form

$$V_t(K_t) = \max_{K_{t+1}} d(K_t, K_{t+1}) + \phi V_{t+1}(K_{t+1}) \qquad t = 0,1,..T \tag{7}$$

$$s.t.\ V_{T+1}(K_{T+1}) = W(K_{T+1}) \qquad (\textit{terminal condition})$$

where $V_t(K_t)$ is the value of the firm at the beginning of period *t* given the capital stock K_t; $V_{t+1}(K_{t+1})$ is the value of the firm at the beginning of the next period. [6]

Proposition 1 characterizes the solution of the problem given in (7).

Proposition 1: *The firm's optimal capital stock* K^*_{t+1} *at the end of each period t in the no-breach scenario is characterized by*

$$F'(K^*_{t+1}) = \frac{1}{\phi} - (1-\beta), \quad t = 0,1,2......T \tag{8}$$

Proof: See Appendix.

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[5] The decisions are equivalently represented by capital allocations $\{k_0, k_1, ..., k_T\}$ in the planning horizon.

[6] The formulation given here assumes that the residual cash flow after investment in capital stock can be invested in non-operating assets that gives expected risk-adjusted returns at the rate of $\left(\frac{1}{\phi} - 1\right)$ which is the same rate that the investors can earn if it is distributed to them as dividends. Further, we assume that the liquidation proceeds of capital stock at the end of the horizon equal its value at that time.

The RHS of (8) is a constant which implies that the optimal capital stock will also be constant over the entire time horizon and we use K^* to denote the constant optimal capital stock.[7] From the above expression K^* is larger when the discount rate is closer to 1 and β is larger, a result that is fairly intuitive.

Corollary: The firm's allocations do not change under a myopic decision rule in which the firm solves the following

$$V_t(K_t) = \max_{K_{t+1}} d(K_t, K_{t+1}) + \phi \cdot W(K_{t+1}) \qquad t = 0,1,..T \qquad (9)$$

In contrast to the dynamic optimization problem in (7), the objective of the myopic decision focuses on current payoff. However, the corollary shows that such a decision rule leads also to the optimal capital investments in the no-breach scenario.

4. SCENARIO WITH INFORMATION SECURITY BREACH COSTS AND FINANCIAL DISTRESS

In this scenario, we introduce the possibility that information security breaches might impose costs in accordance with the Pareto distribution given in (3). In response, the firm makes allocations s_t to security assuring processes from the cash flow generated by revenue generating processes. Larger allocations to security assuring processes lower the expected breach costs and the probability of financial distress. But it also reduces the funds available for allocation to current and future revenue generating processes and building capital stock.

When the information security breach costs are higher than the available internal funds[8] the firm resorts to borrowing from the capital market. This results in

[7] In the initial periods, it is possible that the first order condition is not satisfied at an interior point. In that case, the entire net cash flow is plowed back to increase the capital stock till the time that the steady state capital stock K^* is reached.

[8] Note that internal funds come from operations and the initial capital that could include borrowed funds. The borrowing that we refer to here is the additional cash infusion in periods t = 1,2,…..T to cover realized security breach costs that exceed internal funds available at that time.

borrowing costs in addition to the breach costs. When the amount of borrowing exceeds the limit D (imposed by the capital market)[9] the firm faces financial distress. We assume that in case of financial distress, the firm continues to operate but faces additional costs that include legal and administrative costs of re-structuring the loans, costs resulting from impaired ability to conduct business, costs of a demoralized workforce as well as agency costs of underinvestment, overinvestment and abandonment (Ross et al. 2008). We now formalize the above arguments regarding breach costs and financial distress.

We denote the available internal fund after investment decisions in period t by $z_t = y_t - k_t - s_t$. Substituting for k_t, we have $z_t = F(K_t) - K_{t+1} + (1-\beta)K_t - s_t$. This internal fund is the amount that is available for covering the costs incurred by information security breaches in period t. The allocation s_t in security assuring processes, results in a distribution of security breach costs with a location parameter $h(s_t)$ as given in (5).

Figure 1 depicts the three mutually exclusive ranges of information breach costs that are relevant for our analysis. In the first range, the breach cost ξ_t is at least $h(s_t)$ but lower than the residual cash flow z_t and therefore it can be fully covered internally. The probability is given as $\Pr(h(s_t) \leq \xi_t < z_t) = (1-\rho_t)$ where $\rho_t = \left(\frac{z_t}{h(s_t)}\right)^{-\delta}$.

In the second range, the breach cost ξ_t lies between z_t and $(D+ z_t)$ in which case the internal residual cash flow alone will not cover the breach costs. Hence, the firm needs to borrow an amount equal to $(\xi_t - z_t)$ at an interest rate of r. The last range is where the breach cost ξ_t exceeds $(D+ z_t)$ resulting in financial distress for the firm.

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[9] We assume that the capital market is not perfect and a borrowing limit exists beyond which the firm incurs incremental financial distress costs.

The probability is given as $Pr(\xi_t > z_t + D)$ = θ_t where $\theta_t = \left(\frac{z_t + D}{h(s_t)}\right)^{-\delta}$. In this range, the costs incurred by the firm comprise two components. The first component is the cost associated with borrowing an amount *D* at an interest rate of *r*. The debt and the interest need to be typically repaid under a restructured agreement over an extended period of time. For simplicity, we assume that they need to be repaid within one period. In effect, the payment will be *D*(*1+r*). The second component is the cost of financial distress which we denote by an amount ϖ that includes all the other financial distress costs. For simplicity we model ϖ as a lump-sum constant amount.

Insert Figure 1 about here

The realized one-period residual cash flow after paying security breach costs at the end of the period t is given as follows:

$$\begin{aligned} &I(z_t \geq \xi_t)\cdot(z_t - \xi_t) - I(z_t < \xi_t \leq D + z_t)\cdot(1+r)\cdot(\xi_t - z_t) \\ &- I(z_t + D < \xi_t)\cdot(D(1+r) + \varpi) \end{aligned} \tag{10}$$

where $I(.)$ represents the indicator function for the respective breach cost ranges. Taking expectation with respect to ξ_t, we get the one-period expected residual cash flow as

$$\pi(K_t, K_{t+1}, s_t) = z_t - \frac{\delta}{\delta - 1}h(s_t) - \frac{r}{\delta - 1}\rho_t z_t - \theta_t\left[(D(1+r) + \varpi) - (1+r)\left(\frac{\delta}{\delta - 1}(z_t + D) - z_t\right)\right] \tag{11}$$

The above expression describes the partial one-period solution within the context of the overall multi-period optimization problem given a beginning-period capital stock, K_t. Even so, we can still obtain some insight by analyzing how the probabilities of borrowing and bankruptcy as well as the expected residual cash flow respond to changes in s_t, given a beginning-period capital stock, K_t. Using numerical illustrations, we find that as the firm increases s_t, both the need to borrow and the probability of financial distress decrease. This is reflected in the relationship between the location parameter $h(s_t)$ and borrowing and financial distress probabilities (ρ_t and θ_t respectively) as shown in Figure 2a. These probabilities also decrease with the shape parameter δ as shown in Figure 2b. Although δ is a constant in our model, having it as an increasing function of s_t would still be consistent with Figure 2b. When s_t is increased, the location parameter $h(s_t)$ decreases and as shown in Figure 2c, the residual cash flow increases. This implies that the expected breach cost reduction from higher s_t more than compensates the potential decrease in cash flow due to lower allocation to revenue generating operations. As expected, the residual cash flow increases when the shape parameter δ increases (shown in Figure 2d).

Insert Figures 2a-2d here

We now present some formal results. Lemmas 1 and 2 develop and solve the allocation problem under the myopic decision rule. Proposition 2 gives the results using dynamic optimization.

Lemma 1: The expected one-period residual cash flow function is concave in both the decision variables (K_{t+1}, s_t) and in the initial capital stock K_t.

Proof: The proof is by direct differentiation. See Appendix for details.

We first look at capital allocations under myopic decisions in which the firm maximizes $\pi(K_t, K_{t+1}, s_t) + \phi \cdot [F(K_{t+1}) + (1-\beta)K_{t+1}]$ for any $t \in [1, T+1]$, and the findings are summarized in Lemma 2.

Lemma 2: Let K_t^m be the optimal capital stock under myopic decision rule. Then for any $t = 1,2,...T+1$, $K_t^m < K^*$ and $K_t^m \to K^*$ if $r = 0$ and $D \to +\infty$.

Proof: See Appendix.

According to Lemma 2, in the presence of borrowing costs and financial distress, the solution under the myopic decision requires the capital stock at the end of any period to be less than the optimal benchmark solution. This result obtains from the needed allocations to security assuring operations which was not necessary in the no-breach scenario.

When the firm chooses allocations to maximize future payoffs, it solves

$$V_0(K_0) = \underset{\{K_{t+1}, s_t\}_{t=0}^{T}}{Max} \sum_{t=0}^{T} \phi^t \pi(K_t, K_{t+1}, s_t) + \phi^{T+1} W(K_{T+1}) \qquad (12)$$

The above formulation is different from (6) in that it captures the effects of borrowing, financial distress and allocation to security. In this case, the firm needs to trade off current information security breach costs against reductions in expected residual cash flows in future periods. In other words, the firm reduces current expected breach costs by diverting allocations from revenue generating to security assuring operations. As a result, future potential for residual cash flow generation could be compromised.

Again, the dynamic optimization problem in (12) can be expressed as the following recursive form

$$V_t(K_t) = \max_{\{K_{t+1}, s_t\}} \pi(K_{t+1}, s_t; K_t) + \phi V_{t+1}(K_{t+1}) \qquad t = 0,1,..T \qquad (13)$$

$$s.t.\ V_{T+1}(K_{T+1}) = W(K_{T+1}) \qquad \text{(terminal condition)}$$

The solution for (13) is characterized in Proposition 2. We first show that an interior solution to (13) exists in Lemma 3.

Lemma 3:

(1) *The value function* $V_t(K_t)$ *for each* $t \in [0,T]$ *is strictly concave.*

(2) *The value function* $V_t(K_t)$ *for each* $t \in [0,T]$ *is differentiable and*

$V_t'(K_t) = \dfrac{\partial \pi(K_t, K_{t+1}^*, s_t^*)}{\partial K_t}$ *for each* $t \in [0,T]$ *, where* K_{t+1}^* *and* s_t^* *denote the optimal allocations in period t.*

Proof: See Appendix

Using the results of Lemma 3, we now proceed to use the first order conditions to characterize the solution in Proposition 2.

Proposition 2: The optimal revenue generating and security assuring allocations are determined uniquely by

$$F'(K_{t+1}) = \frac{H_t(K_t, K_{t+1}, s_t)}{\phi H_{t+1}(K_{t+1}, K_{t+2}, s_{t+1})} - (1-\beta), \quad t = 0,1,...T-1 \tag{14}$$

$$F'(K_{T+1}) = \frac{H_T(K_T, K_{T+1}, s_T)}{\phi} - (1-\beta) \tag{15}$$

$$h'(s_t) = \frac{H_t(K_t, K_{t+1}, s_t)}{\Psi_t(K_t, K_{t+1}, s_t)}, \quad t = 0,1,...T \tag{16}$$

Where

$$H_t(K_t, K_{t+1}, s_t) = 1 + r\rho_t + \theta_t\left\{\frac{1+r}{\delta-1} + \frac{\delta}{z_t+D}\left[(D(1+r)+\varpi) - (1+r)\left(\frac{\delta}{\delta-1}(z_t+D) - z_t\right)\right]\right\}$$

and

$$\Psi_t(K_t, K_{t+1}, s_t) = -\frac{\delta\theta_t}{h(s_t)}\left[(D(1+r)+\varpi) - (1+r)\left(\frac{\delta}{\delta-1}(z_t+D) - z_t\right)\right] - \frac{\delta}{\delta-1}(1+r\eta_t)$$, in

which $\eta_t = \left(\frac{z_t}{h(s_t)}\right)^{-\delta+1}$

Proof: See Appendix.

Equations (14) – (16) are nonlinear second-order difference equations characterizing the optimal allocations in the planning horizon. Equation (15) indicates that at the end of terminal period T, the optimal allocations are the same as those under the myopic decision rule. The terminal period capital stock K_{T+1} can be shown (by examining the sign of the derivative of $F'(K_{T+1})$) to be higher when the financial distress cost ϖ is smaller and/or the borrowing limit D is larger. From (16), it can be shown that s_t is lower when ϖ is smaller, an intuitive result. The three equations provide *2T* equations to solve the *2T* allocations $(K_1, K_2,..., K_{T+1}, s_0, s_1,..., s_T)$ uniquely. Now we proceed to develop the steady-state behavior of the capital stock under borrowing and financial distress costs.

Proposition 3: The steady-state capital stock determined by equations (14) and (15) is the same as the optimal no-breach capital stock (K^*).

Proof: See Appendix.

In steady state, the capital stock and the allocation to security assuring operations remain constant over time. Given that K_t and s_t vary monotonically over time, the steady state can only be achieved asymptotically in the long run. According to Proposition 3, the steady-state capital stock is the same as the one under the no-breach scenario.

Recall that the capital stock K^* under the no-breach scenario is not affected by parameters $\Delta = (D, \varpi, r)$. Therefore it follows from the proposition that the borrowing and financial distress costs do not affect the long run steady state capital stock. The intuition here is that a firm needs to protect itself from financial distress caused by both current and future breach costs. In order to mitigate the chance of future breach costs causing financial distress, the firm accumulates capital stock by allocating more resources to revenue generating operations. If the borrowing and financial distress costs are high (low), the firm requires a long (short) time to accumulate the capital stock necessary to cover future expected financial distress costs. Likewise, when a firm is endowed with low (high) initial capital stock, it requires a long (short) time to accumulate the capital stock. In all cases, the capital stock will approach the steady-state capital stock of the no-breach scenario.

Numerical Illustration

Now, we numerically explore the firm's allocations to revenue generating and security assuring operations over time. The revenue generating function used in the illustration is Cobb-Douglas where $F(K_t) = 100K_t^{0.3}$ and the security assuring function is characterized by $h(s_t) = 20 + 100 \cdot (1 + s_t)^{-0.1}$. The planning horizon is assumed to be T = 500. The parameters held constant are $\beta = 0.15$, interest rate r =0.05 and discount rate $\phi = 0.995$. The following parameters are changed: (i) debt limit D = 300, 500, 1000; (ii) aggregate financial distress cost $\varpi = 10000, 20000, 30000$; (iii) shape parameter $\delta = 2.1, 3.0, 5.0$.

Figures 3a to 3c depict capital stock accumulations and Figures 3d to 3f give security allocations over the first 20 time periods of the planning horizon. Under the chosen values of the parameters, the no-breach steady state capital stock (K^*) is about 1848. Starting from an initial capital stock (K_0=750), the firm accumulates capital stock asymptotically to K^* over the planning horizon. Initially, when capital stock is low, the firm relies more on higher security allocations to cover current breach costs. As the firm's capital stock approaches the steady-state level, s_t drops. The illustration also shows that a firm accumulates capital stock faster if the borrowing limit is higher and/or the aggregate financial distress cost and/or the uncertainty is lower. Further, s_t is higher if D is lower and/or the aggregate financial distress cost and/or the uncertainty is higher.

5. SCENARIO WITH INFORMATION SECURITY BREACH COSTS, FINANCIAL DISTRESS AND INSURANCE

In this section, we examine the case where the firm has the opportunity to purchase external insurance which allows it to claim part of the realized information breach costs. Note that both the firm and the insurance provider are modeled as being risk neutral. In other words, risk aversion is not a necessary condition for our results. Further, note that the insurance premium paid by the firm is not included in the breach costs. However, this would be the case if one were to develop a socially optimum allocation to security investments (see Kesan et al. 2005).

We use $\sigma_t \equiv \mu\xi_t$ (where $\mu \in (0,1)$) to denote the firm's net realized information breach costs after claiming the insurance. Thus ($1-\mu$) represents the external insurance coverage obtained by the firm. We denote the insurance premium paid by the firm as $\tau(\mu)$ which is a decreasing function of μ.

Since ξ_t follows the Pareto distribution, we get the density function of σ_t as

$$f(\sigma_t) = \frac{1}{\mu} \frac{\delta \cdot h(s_t)^{\delta}}{\left(\frac{1}{\mu}\sigma_t\right)^{\delta+1}} = \frac{\delta \cdot (\mu \cdot h(s_t))^{\delta}}{\sigma_t^{\delta+1}} \qquad (17)$$

Equation (17) shows that σ_t also follows a Pareto distribution with shape parameter δ and location parameter $\mu \cdot h(s_t)$. The optimal allocations with this insurance coverage can be determined uniquely by equations (14) - (16) after replacing ρ_t with

$\tilde{\rho}_t \equiv \left(\frac{z_t}{\mu h(s_t)}\right)^{-\delta}$ and θ_t with $\tilde{\theta}_t \equiv \left(\frac{z_t + D}{\mu h(s_t)}\right)^{-\delta}$.

Proposition 4: In any period t, insurance with a premium $\tau(\mu)$ is feasible if the following condition holds:

$$\frac{(1-\mu)}{(1-\mu^{\delta})} < \frac{\theta_t}{h(s_t)}\left[\frac{\delta}{(\delta-1)}\varpi + z_t + D\right] \qquad (18)$$

Proof: Using (17) and the definitions of $\tilde{\rho}_t$ and $\tilde{\theta}_t$, the decrease in the expected cost of financial distress to the firm when it purchases insurance can be shown to be

$$\Delta EFD = (1-\mu^{\delta})\frac{h(s_t)^{\delta}}{(D+z_t)^{(\delta-1)}}\left[\frac{1}{(D+z_t)} + \frac{\delta}{(\delta-1)}\right]$$

The firm will find it advantageous to purchase insurance in period t if $\Delta EFD > \tau(\mu)$.

The expected payout by the insurance provider is $(1-\mu)\frac{\delta h(s_t)}{(\delta-1)}$. The insurance provider will offer the insurance if its expected payout is less than the premium $\tau(\mu)$. Following the above reasoning, we get (18). ■

We now illustrate Proposition 4 by a numerical example where ϖ = 200000, D = 1000, z_t = 500 and $h(s_t)$ = 40. Figure 4a shows the effect of variation in the shape parameter δ on the range (RHS – LHS) of the inequality (15). When this range is positive, purchasing insurance can be beneficial to both the firm and the insurance provider. This is true for smaller values of δ. For larger values of δ, the probability of financial distress and the resulting cost is smaller. This could render the purchase of insurance unnecessary. Figure 4b shows the effect of the variation in μ (1 – coverage) on the range. Consistent with our intuition, the plot shows that there is a threshold level beyond which the coverage becomes excessive (i.e., the range becomes negative).

Insert Figures 4a-4b here

We now formalize the effect of insurance coverage on the accumulation of capital stock in the following proposition.

Proposition 5: Insurance coverage has no effect on the steady-state capital stock.

Proof: From equation (11), it can be seen that insurance coverage affects only $H(K_t, K_{t+1}, s_t)$, and such effects are canceled out at the steady-state because $H(K_t, K_{t+1}, s_t) = H(K_{t+1}, K_{t+2}, s_{t+1})$ when $K_t = \overline{K}$ and $s_t = \overline{s}$ for all *t*. ■

We illustrate the effect of coverage on the capital stock and allocation to security assuring operations in Figures 5a and 5b where the revenue generation function is Cobb-Douglas $F(K_t) = 100K_t^{0.3}$, the security assuring function is $h(s_t) = 20 + 100 \cdot (1 + s_t)^{-0.1}$ and other relevant parameters are $\beta = 0.15$, $r = 0.05$,

$\phi = 0.995$, $\delta = 2.1$, $D = 500$, and $\varpi = 10000$. The planning horizon is assumed to be T = 500 out of which the plots cover the first 20 time periods. Figure 5a shows that insurance coverage has no effect on the steady-state capital stock. Figure 5b shows that s_t decreases when insurance coverage (1- μ) increases. An implication of these two effects is that when the planning horizon is long, the firm can use insurance strategically in the initial periods to speed up capital stock accumulation.

Insert Figures 5a-5b here

6. SUMMARY AND CONCLUDING REMARKS

The Computer Security Institute (CSI) in its 2007 survey, reports that the average annual loss due to information security breaches has more than doubled, from $168,000 in 2006 to $350,424. This highlights the need for a firm to effectively manage the security breach costs. At a broader level, the firm allocates funds between revenue generating and security assuring processes. At the next level, the focus shifts to using the allocation from the first level to assess and manage individual security technologies. In this paper, we examine the allocations to revenue generating and security assuring operations from a broader organizational perspective in the presence of costly security breaches that could result in financial distress. In addition, we also investigate the role that external insurance could play in mitigating the effects of breach costs.

Our models and supporting numerical analyses dealing with the allocation of resources to security assurance operations provide the following results. At the firm level, the optimal revenue generating allocation is lower in the initial periods when security breach, borrowing and financial distress costs are higher. In particular, when the firm's planning horizon is short, this results in a lower investment in revenue generating activities when breach-related costs are higher. Further, the reduction in capital stock build-up compared to no-breach case persists longer when the financial distress costs are higher and/or when the debt limit is lower. In the long run, however, the steady state capital stock accumulation is the same as in the no-breach case even in the presence of financial distress and borrowing costs. Primarily, security breach costs affect the rate of capital build up and the residual cash flow from revenue generating activities that is available to the investor for further investment and consumption. These findings highlight the effect of security breach costs on the growth rates of firms and suggest that sectors of the economy that have a higher propensity for security breaches might exhibit relatively lower growth rates. These findings also highlight the importance of developing systems that collect and classify security breach costs and organizational mechanisms to integrate this data with financing constraints known to the corporate finance department. Such data is useful in estimating the distribution of the breach costs and applying the models developed here for optimal allocation between revenue generating and security assurance operations.

Our investigation of the role of external insurance indicates the existence of a range of breach costs over which insurance could be beneficial to both the firm and the provider. We also identify the relevant parameter values that yield such a range. Furthermore, even though our analysis shows that insurance *per se* does not affect the steady state allocations, it helps firms with short planning horizons to speedily accumulate capital stock. Insurance covering a long planning horizon however, has little effect on capital stock accumulation. These findings also have important accounting policy implications. Writing in Inc.com in April 2007, Dan Briody comments as follows: "*But purchasing a cyber insurance policy is far from a no-brainer. The policies are often confusing and pricey. The main problem: Cyber risk* *has been frustratingly difficult for insurers to quantify."* Quantification of insurance policies requires the assessment of the overall distribution of security breach costs across firms by insurance companies. Disclosure of security costs by firms can be mandated and made credible through the financial accounting system. Such a system could go a long way in identifying efficient insurance opportunities and result in a welfare gain.

In order to counter increasingly complex security breach problems, an organization needs both improved technological solutions and effective managerial approaches regarding planning and allocation of resources. This paper highlights the importance of resource allocation and insurance at the organizational level in addressing security breach problems. In particular, the methodology employed in this paper enables managers to seek and use relevant information in an informed manner.

While this analysis has focused on information security breaches, it can be broadened to address resource allocation in the presence of other security and internal control breaches (for example, terrorist attacks, governance failures etc.). An important assumption made in the paper is that the managers take decisions in the interest of the firm and its investors. However, if the mangers have a shorter planning horizon compared to the investors, this could result in excessive allocation to security assuring operations to reduce the probability of job loss in the event of financial distress. This provides an opportunity for a future study to explore this agency effect in the context of overlapping generations of managers. The analysis as carried out does not account for potential learning about the effectiveness of the resources allocated to security in the previous period. A future study could also incorporate this learning effect in fine-tuning the allocations over the planning horizon.

APPENDIX

Proof of Proposition 1: Let K_t^* denote the optimal investment decision made at the beginning of time period t. We show that if K_{t+1}^* is a constant (not a function of K_t^*), so is K_t^* and K_t^* is determined by $F'(K_t)=\frac{1}{\phi}-(1-\beta)$.

The value function at time t is

$$V_t(K_t)=F(K_t)-K_{t+1}^*+(1-\beta)K_t+\phi V_{t+1}(K_{t+1}^*)$$

Differentiating, we have $V_t'(K_t)=F'(K_t)+(1-\beta)$ if K_{t+1}^* is a constant.
The solution is then determined by the first order condition

$\phi V_t'(K_t)=1\Rightarrow F'(K_t)=\frac{1}{\phi}-(1-\beta)$, and K_t^* is a constant.

We can then finish the proof by showing that K_{T+1}^* is a constant and is determined by $F'(K_{T+1})=\frac{1}{\phi}-(1-\beta)$. At $t=T$, the firm solves

$$\underset{K_{T+1}}{Max}\, F(K_T)-K_{T+1}+(1-\beta)K_T+\phi F(K_{T+1})+\phi(1-\beta)K_{T+1}$$

So K_{T+1}^* is a constant and it is determined by the first order condition $F'(K_{T+1})=\frac{1}{\phi}-(1-\beta)$. ■

Proof of Lemma 1: Define $M_1(K_t,K_{t+1},s_t)=z_t-\frac{\delta}{\delta-1}h(s_t)-\frac{r}{\delta-1}\rho_t z_t$ and $M_2(K_t,K_{t+1},s_t)=\theta_t\left[(1+r)\left(\frac{\delta}{\delta-1}(z_t+D)-z_t\right)-(D(1+r)+\varpi)\right]=\theta_t\cdot E_t$, the one-period residual cash flow is

$$\pi(K_t,K_{t+1},s_t)=M_1(K_t,K_{t+1},s_t)+M_2(K_t,K_{t+1},s_t)$$

We can show that both $M_1(K_t,K_{t+1},s_t)$ and $M_2(K_t,K_{t+1},s_t)$ are concave in decision variables and state parameter. First for $M_1(K_t,K_{t+1},s_t)$, by direct differentiation with respect to K_t ,

$$\frac{\partial M_1(K_t,K_{t+1},s_t)}{\partial K_t}=[F'(K_t)+(1-\beta)]\cdot(1+r\cdot\rho_t)>0$$

$$\frac{\partial^2 M_1(K_t, K_{t+1}, s_t)}{\partial K_t^2} = F''(K_t)[1 + r \cdot \rho_t] - r[F'(K_t) + (1-\beta)]\frac{\partial \rho_t}{\partial K_t} < 0$$

Then, by direct differentiation respect to the decision variables,

$$G_K(K_{t+1}, s_t) \equiv \frac{\partial M_1(K_t, K_{t+1}, s_t)}{\partial K_{t+1}} = -1 - r \cdot \rho_t$$

$$G_s(K_{t+1}, s) \equiv \frac{\partial M_1(K_t, K_{t+1}, s_t)}{\partial s_t} = -\frac{\delta}{\delta - 1} h'(s_t) \cdot \left(1 + r \cdot \left(\frac{z_t}{h(s_t)}\right)^{1-\delta}\right) - 1 - r\rho_t$$

The Hessian matrix is then

$$H1 = \begin{bmatrix} G_{KK} & G_{Ks} \\ G_{sK} & G_{ss} \end{bmatrix}$$

Where

$$G_{KK} \equiv \frac{\partial G_K(\cdot)}{\partial K_{t+1}} = -\delta \cdot r \cdot \rho_t \cdot \frac{1}{z_t}$$

$$G_{Ks} \equiv \frac{\partial G_K(\cdot)}{\partial s_t} = G_{sK} \equiv \frac{\partial G_s(\cdot)}{\partial K_{t+1}} = -\delta \cdot r \cdot \rho_t \cdot \left(\frac{h'(s_t)}{h(s_t)} + \frac{1}{z_t}\right)$$

$$G_{ss} \equiv \frac{\partial G_s(\cdot)}{\partial s_t} = \frac{\delta}{1-\delta} \cdot h''(s_t) \cdot \left(1 + r \cdot \left(\frac{z_t}{h(s_t)}\right)^{1-\delta}\right) - 2 \cdot \delta \cdot r \cdot \rho_t \cdot \frac{h'(s_t)}{h(s_t)} - \delta \cdot r \cdot \rho_t \cdot \left(\frac{h'(s_t)}{h(s_t)}\right)^2 z_t$$
$$- \delta \cdot r \cdot \rho_t \cdot z_t^{-1}$$

The determinant of H1 is $det(H1) = \frac{\delta}{\delta - 1} h''(s_t) \cdot \left[1 + \left(\frac{z_t}{h(s_t)}\right)^{1-\delta}\right] \cdot \delta \cdot r \cdot \rho_t \cdot \frac{1}{z_t} > 0$.

Combining this with $G_{KK} < 0$, we get that $M_1(K_{t+1}, s_t; K_t)$ is strictly concave in the decision variables.

For $M_2(K_t, K_{t+1}, s_t)$, by direct differentiation with respect to K_t, we get

$$\frac{\partial M_2(K_t, K_{t+1}, s_t)}{\partial K_t} = -\frac{\delta}{z_t + D} \theta_t \cdot E_t \cdot [F'(K_t) + (1-\beta)] + \theta_t \frac{1+r}{\delta - 1}[F'(K_t) + (1-\beta)]$$

$$\frac{\partial^2 M_2(K_t,K_{t+1},s_t)}{\partial K_t^2} = \left(\frac{\delta\theta_t}{z_t+D}\left[F'(K_t)+(1-\beta)\right]\right)^2 \cdot E_t + \frac{\delta\theta_t}{(z_t+D)^2}\left[F'(K_t)+(1-\beta)\right]\cdot E_t$$
$$-\frac{\delta\theta_t}{z_t+D}F''(K_t)\cdot E_t - 2\frac{\delta\theta_t}{z_t+D}\frac{1+r}{\delta-1}\left[F'(K_t)+(1-\beta)\right]+\theta_t\frac{1+r}{\delta-1}F''(K_t)<0$$

Above term is strictly negative because $E_t < 0$ and $F(K_t)$ is increasing and concave.

By direct differentiation with respect to decision variables,

$$\frac{\partial M_2(K_t,K_{t+1},s_t)}{\partial K_{t+1}} = \frac{\delta}{z_t+D}\theta_t E_t - \theta_t\frac{1+r}{\delta-1}$$

$$\frac{\partial M_2(K_t,K_{t+1},s_t)}{\partial s_t} = \delta\cdot\theta_t\cdot\left[\frac{1}{z_t+D}+\frac{h'(s_t)}{h(s_t)}\right]\cdot E_t - \theta_t\frac{1+r}{\delta-1}$$

The Hessian matrix is $H2 = \begin{bmatrix} G_{KK} & G_{Ks} \\ G_{Ks} & G_{ss} \end{bmatrix}$

where

$$G_{KK} \equiv \frac{\partial^2 M_2(K_t,K_{t+1},s_t)}{\partial K_{t+1}^2} = \frac{\delta(1+\delta)\theta_t}{(z_t+D)^2}E_t - 2\frac{\delta\theta_t(1+r)}{(z_t+D)(\delta-1)}<0$$

$$G_{Ks} \equiv \frac{\partial^2 M_2(K_t,K_{t+1},s_t)}{\partial K_{t+1}\partial s_t} = \frac{\delta(1+\delta)\theta_t}{(z_t+D)^2}E_t - 2\frac{\delta\theta_t(1+r)}{(z_t+D)(\delta-1)} + \delta\theta_t(1+r)\frac{h'(s_t)}{h(s_t)} = G_{KK} + \delta\theta_t(1+r)\frac{h'(s_t)}{h(s_t)}$$

$$G_{ss} \equiv \frac{\partial^2 M_2(K_t,K_{t+1},s_t)}{\partial s_t^2} = G_{KK} + \delta\theta_t(1+r)\frac{h'(s_t)}{h(s_t)}\frac{D}{z_t+D} + \delta\theta_t\frac{h''(s_t)}{h(s_t)}E_t<0$$

Thus

$$\det(H2) = \delta\theta_t(1+r)\frac{h'(s_t)}{h(s_t)}\cdot\left[2G_{KK} + G_{KK}\frac{D}{z_t+D} + \delta\theta_t(1+r)\frac{h'(s_t)}{h(s_t)}\right] + \delta\theta_t(1+r)\frac{h''(s_t)}{h(s_t)}E_t G_{KK}>0$$

and $M_2(K_t,K_{t+1},s_t)$ is concave in decision variables. ■

Proof of Lemma 2: Under myopic decision rule, the firm solves

$$\underset{K_{t+1},s_t}{Max} \quad \pi(K_t, K_{t+1}, s_t) + \phi \cdot [F(K_{t+1}) + (1-\beta)K_{t+1}]$$

in each period, and the solutions are uniquely determined by the first-order conditions (by the concavity of both $\pi(K_t, K_{t+1}, s_t)$ and $F(K_{t+1})$):

$$F'(K_{t+1}) = \frac{H_t(K_t, K_{t+1}, s_t)}{\phi} - (1-\beta) \qquad t = 0,1,..T \tag{A.1}$$

$$h'(s_t) = \frac{H_t(K_t, K_{t+1}, s_t)}{\Psi_t(K_t, K_{t+1}, s_t)} \; t = 0,1,..T \tag{A.2}$$

where

$$H_t(K_t, K_{t+1}, s_t) = 1 + r\rho_t + \theta_t \left\{ \frac{1+r}{\delta - 1} + \frac{\delta}{z_t + D} \left[\left(D(1+r) + \varpi \right) - (1+r)\left(\frac{\delta}{\delta - 1}(z_t + D) - z_t \right) \right] \right\}$$

$$\Psi_t(K_t, K_{t+1}, s_t) = -\frac{\delta\theta_t}{h(s_t)} \left[\left(D(1+r) + \varpi \right) - (1+r)\left(\frac{\delta}{\delta - 1}(z_t + D) - z_t \right) \right] - \frac{\delta}{\delta - 1}(1 + r\eta_t)$$

with $\rho_t = \left(\frac{z_t}{h(s_t)} \right)^{-\delta}$, $\theta_t = \left(\frac{z_t + D}{h(s_t)} \right)^{-\delta}$, and $\eta_t = \left(\frac{z_t}{h(s_t)} \right)^{-\delta+1}$.

Recall that $z_t = F(K_t) - K_{t+1} + (1-\beta)K_t - s_t$. Because $H_t(K_t, K_{t+1}, s_t) > 1$ for each given $r > 0$ or $\theta_t > 0$, from the strict concavity of $F(K_{t+1})$ we know that the solution determined by (A.1) is less than K^* which is the steady state of the benchmark scenario. When $r = 0$ and $D \to +\infty$ the solution approaches K^* because $H_t(K_t, K_{t+1}, s_t) \to 1$. ■

Proof of Lemma 3: The proof of Lemma 3: (1) is carried out by backward induction. We first show that $V_T(K_T)$ is strictly concave.

Let $G(K_T, K_{T+1}, s_T) = \pi(K_T, K_{T+1}, s_T) + \phi W(K_{T+1})$, and $G(K_T, K_{T+1}, s_T)$ is concave in both decision variables $\Gamma_T \equiv (K_{T+1}, s_T)$ and the state variable K_T from Lemma 1 and the concavity assumption of the revenue generating function $F(K_{T+1})$.

Considering two capital stocks K'_T and K''_T, let $\Gamma'_T \equiv \Gamma(K'_T)$ and $\Gamma''_T \equiv \Gamma(K''_T)$ denote the optimal solutions corresponding to the two capital stocks; also, let

$K^{\lambda} = (1-\lambda)K'_T + \lambda K''_T$ and $\Gamma^{\lambda} = (1-\lambda)\Gamma'_T + \lambda\Gamma''_T$ for $\lambda \in (0,1)$, we have

$$\begin{aligned} V_T(K^{\lambda}) &= G(\theta(K^{\lambda}), K^{\lambda}) \geq G(\theta^{\lambda}, K^{\lambda}) > (1-\lambda)G(\Gamma'_T, K'_T) + \lambda G(\Gamma''_T, K''_T) \\ &= (1-\lambda)V_T(K'_T) + \lambda V_T(K''_T) \end{aligned}$$

that is, $V_T(K_T)$ is strictly concave.

Defining $G(K_{t-1}, K_t, s_{t-1}) = \pi(K_{t-1}, K_t, s_{t-1}) + \phi V_t(K_t)$, by the same way we can show that if $V_t(K_t)$ is strictly concave, $V_{t-1}(K_{t-1})$ is also strictly concave and this completes proof of (1).

For Lemma 3: (2), consider a capital stock K'_t. Let K^*_{t+1} and s^*_t denote the optimal solutions corresponding to this capital stock. Define

$$L(K_t) = \pi(K_t, K^*_{t+1}, s^*_t) + \phi V_{t+1}(K^*_{t+1})$$

$L(K_t)$ is concave and differentiable; also $L(K_t) \leq V_t(K_t)$ with equality only at $K_t = K'_t$. We then have $V_t(K_t)$ differentiable at K'_t, and $V'_t(K'_t) = L'(K'_t)$ (From Theorem 4.10 in Stokey et al. 1989). ■

Proof of Proposition 2:

The first order conditions of the optimization problem in (10) are:

$$\frac{\partial \pi(K_t, K_{t+1}, s_t)}{\partial K_{t+1}} + \phi V'(K_{t+1}) = 0 \qquad t = 0,1,..T$$

$$\frac{\partial \pi(K_t, K_{t+1}, s_t)}{\partial K_{t+1}} + \phi W'(K_{t+1}) = 0, t = T$$

$$\frac{\partial \pi(K_t, K_{t+1}, s_t)}{\partial s_t} = 0$$

Based on Lemma 2, $V'_t(K_{t+1})$ can be replaced by $\frac{\partial \pi(K_t, K^*_{t+1}, s^*_t)}{\partial K_t}$. Furthermore, the optimal investment plans are determined uniquely by the first-order conditions because of the strict concavity of the value function and the one-period expected profit function. This completes the proof of proposition 2. ■

Proof of Proposition 3:

Let $K_t = \overline{K}$ for each $t \le T$, from (14) security allocation is determined by

$$h'(s_t) = \frac{H_t(\overline{K}, s_t)}{\Psi_t(\overline{K}, s_t)}$$

Since capital stock is fixed, the solution for security allocation will be the same for all periods, that is, $s_t = \overline{s}$ for each t. Now, $H_t(K_t, K_{t+1}, s_t) = H_{t+1}(K_{t+1}, K_{t+2}, s_{t+1})$ from equation (12). The capital stock in each period t is determined by $F'(K_{t+1}) = \frac{1}{\phi} - (1 - \beta)$, which is the same as benchmark steady-state condition. Parameters $\Delta \equiv (D, \varpi, r)$ have no impact on the steady-state capital stock, because they appear only in $H_t(K_t, K_{t+1}, s_t)$, which cancels out in equation (12) under the steady state $(\overline{K}, \overline{s})$. ■

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Table 1: Notations

Parameter	Description
K_t	Capital stock at the beginning of period t
k_t	New investment in revenue generating operations in period t
y_{t+1}	Net cash revenue in period t+1
β	The proportion of the capital stock that is used for replacement and maintenance of capital assets
ξ_t	Information security breach cost, a random variable
s_t	Allocation to Security assuring operations
ς	Rate used to discount future cash flows
z_t	Residual cash flow in period t after allocations to revenue generating and security assuring operations
D	Borrowing Limit
ϖ	Aggregate financial distress cost
r	Interest rate on borrowing
$(1-\mu)$	External insurance coverage
$\tau(\mu)$	External insurance premium for coverage $(1-\mu)$

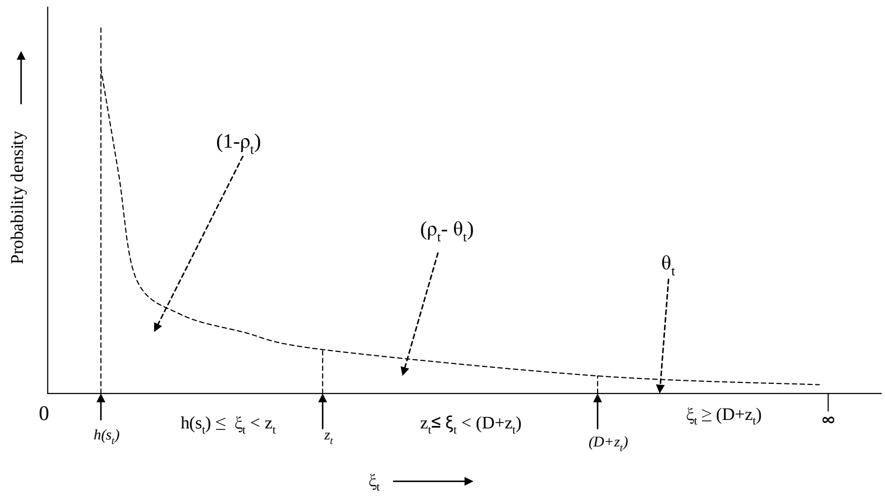

Figure 1: Probability ranges for Information Security Breach Cost ξ_t

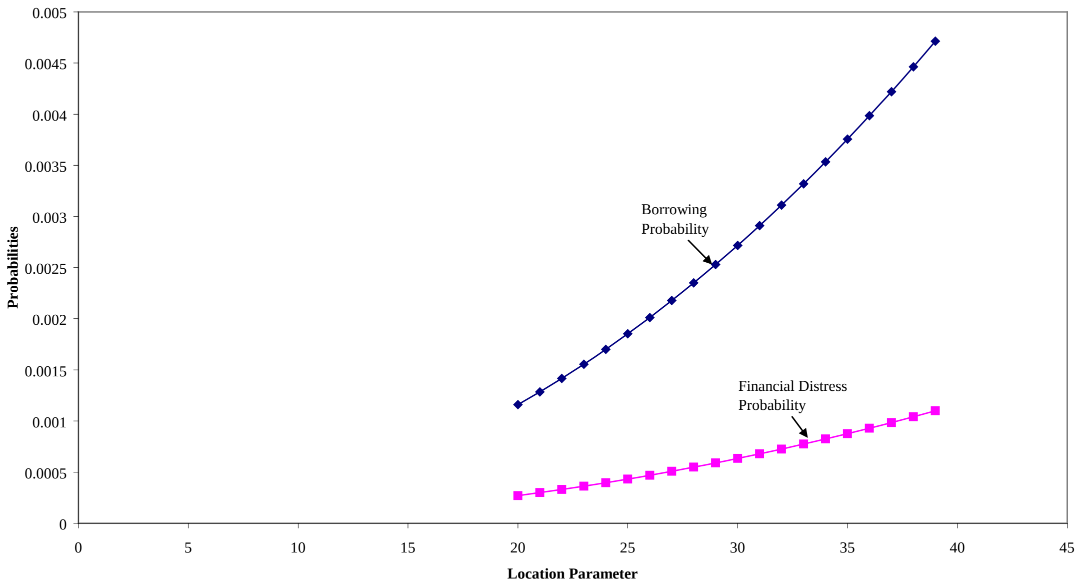

Figure 2a: Borrowing and Financial Distress Probabilities - Location Parameter

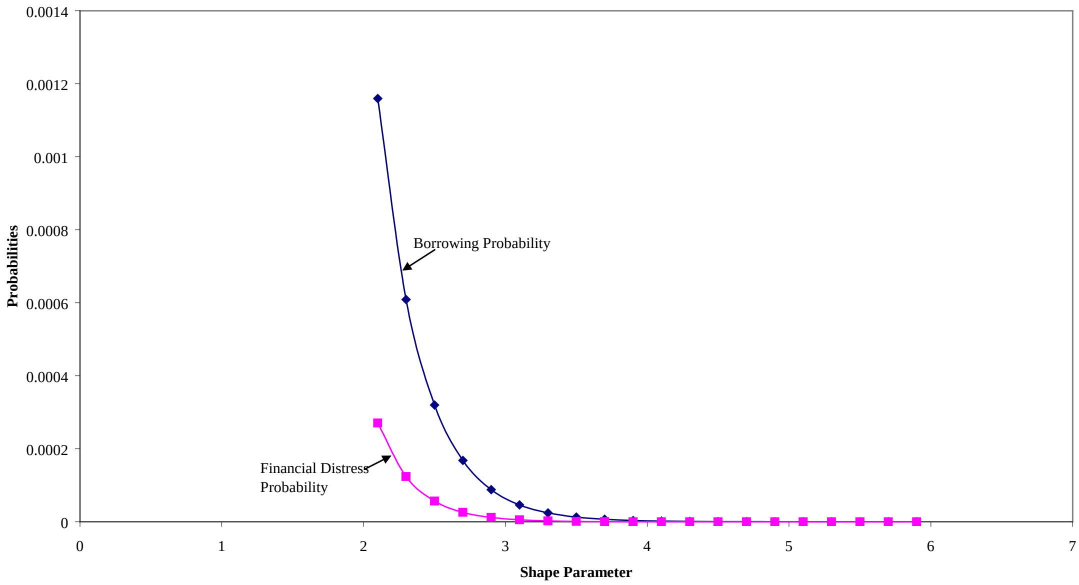

Figure 2b: Borrowing and Financial Distress Probabilities - Shape Parameter

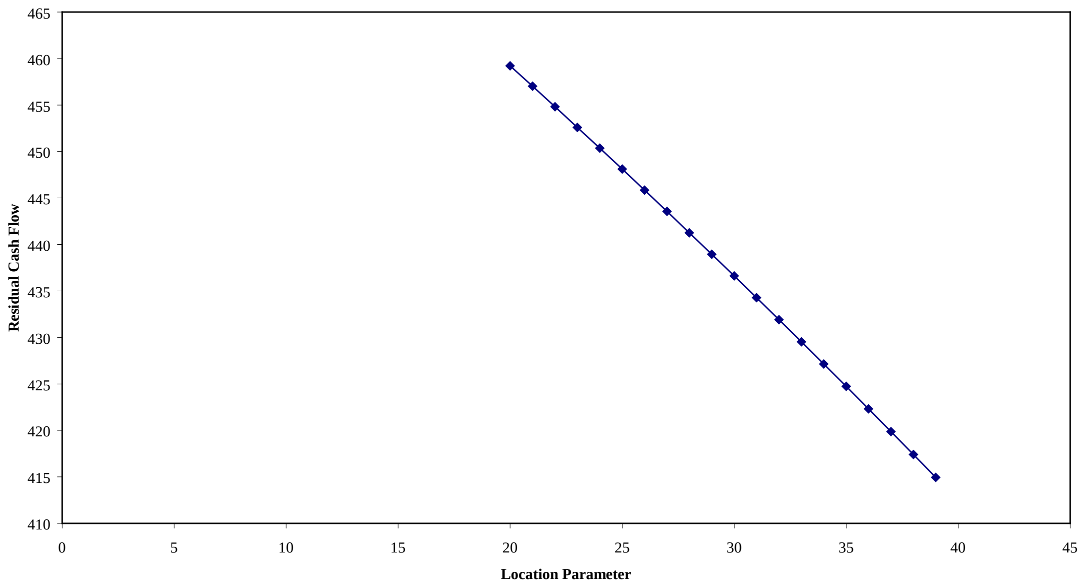

Figure 2c: Residual Cash Flow and Location Parameter

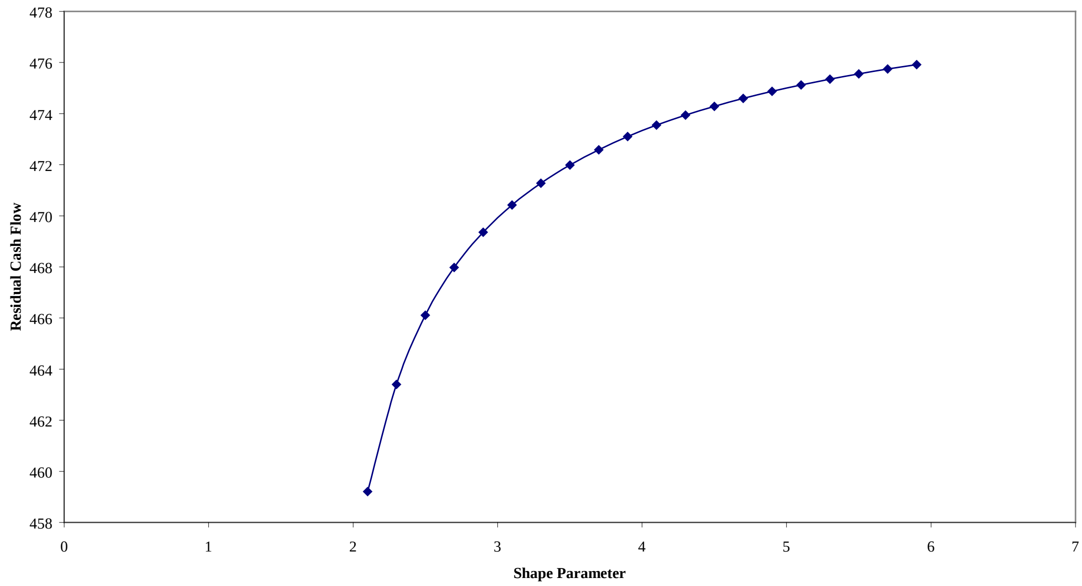

Figure 2d: Residual Cash Flow and Shape Parameter

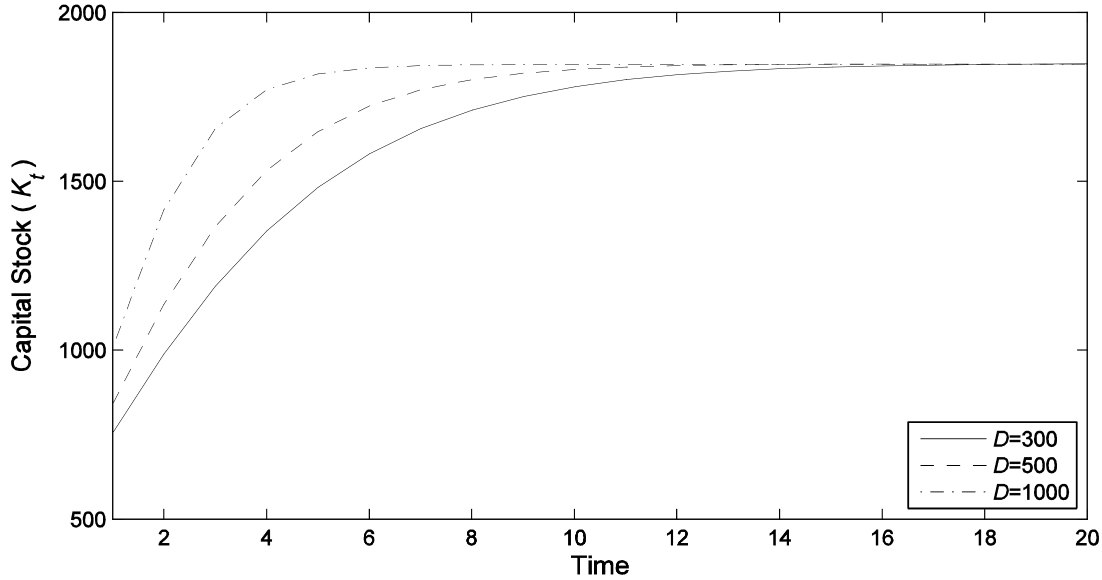

Figure 3a: Capital Stock Accumulation over Time- Different Levels of Debt Limit

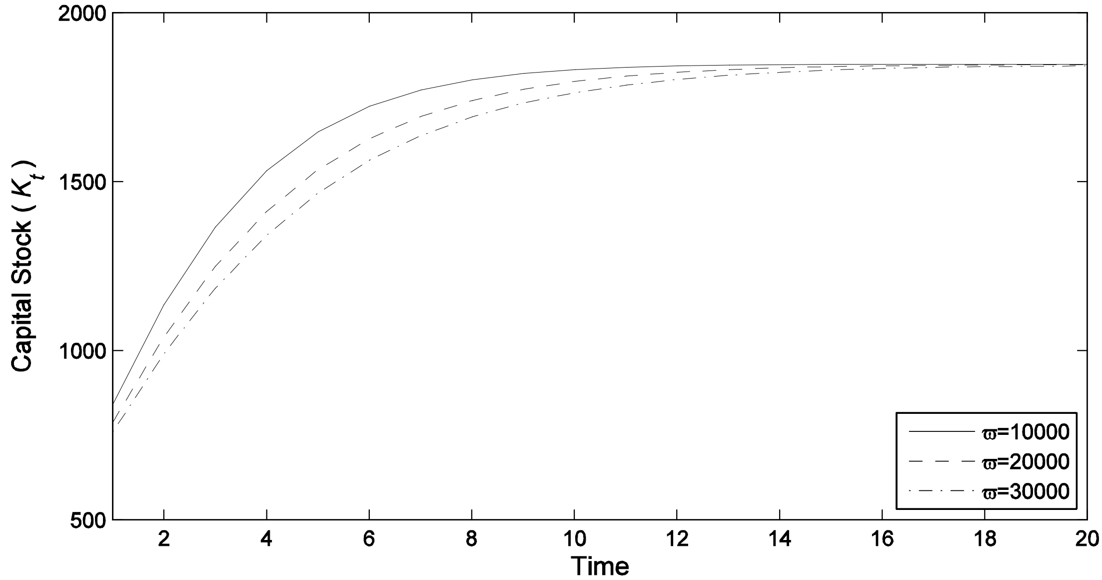

Figure 3b: Capital Stock Accumulation over Time- Different Levels of Financial Distress Cost

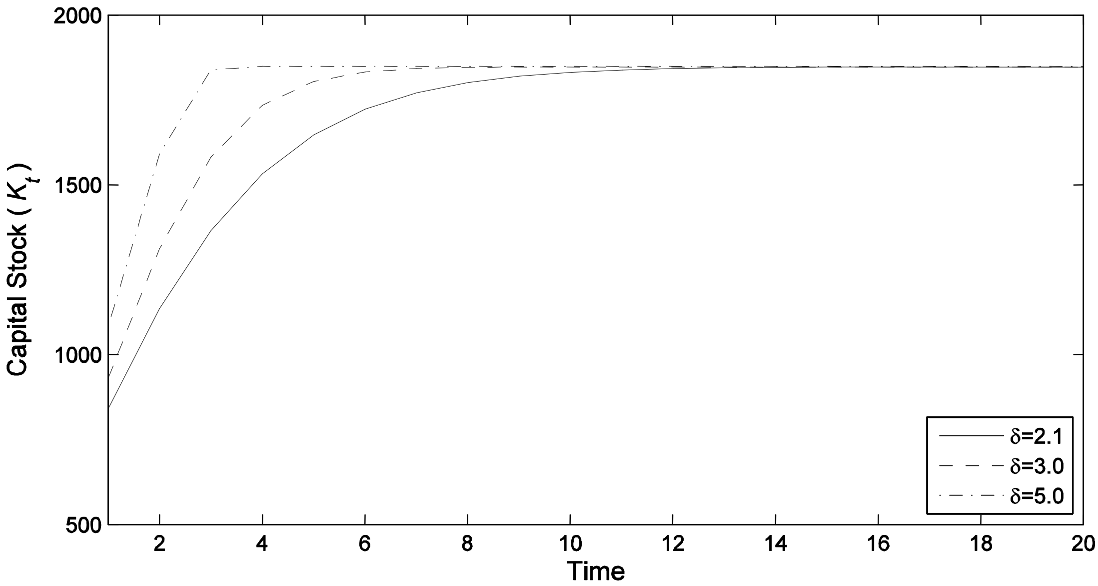

Figure 3c: Capital Stock Accumulation over Time- Different Values of Shape Parameter

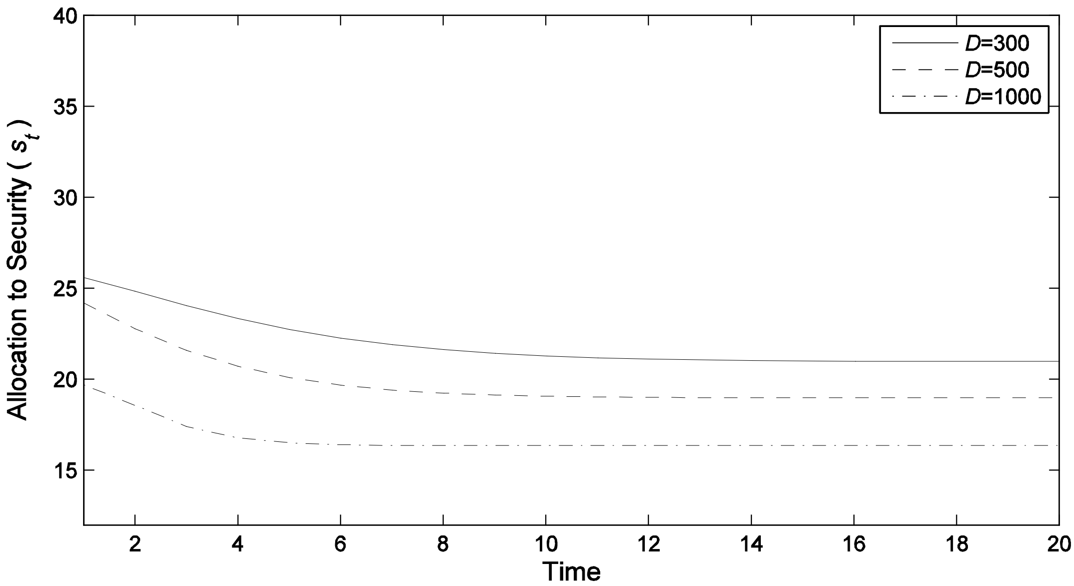

Figure 3d: Allocations to Security over Time- Different Levels of Debt Limit

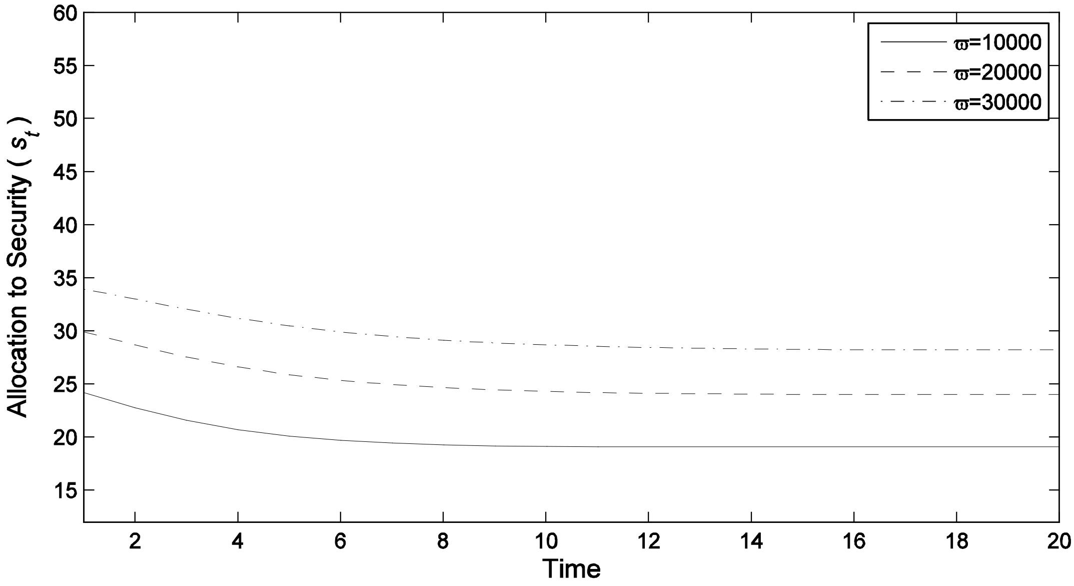

Figure 3e: Allocations to Security over Time - Different Levels of Financial Distress Cost

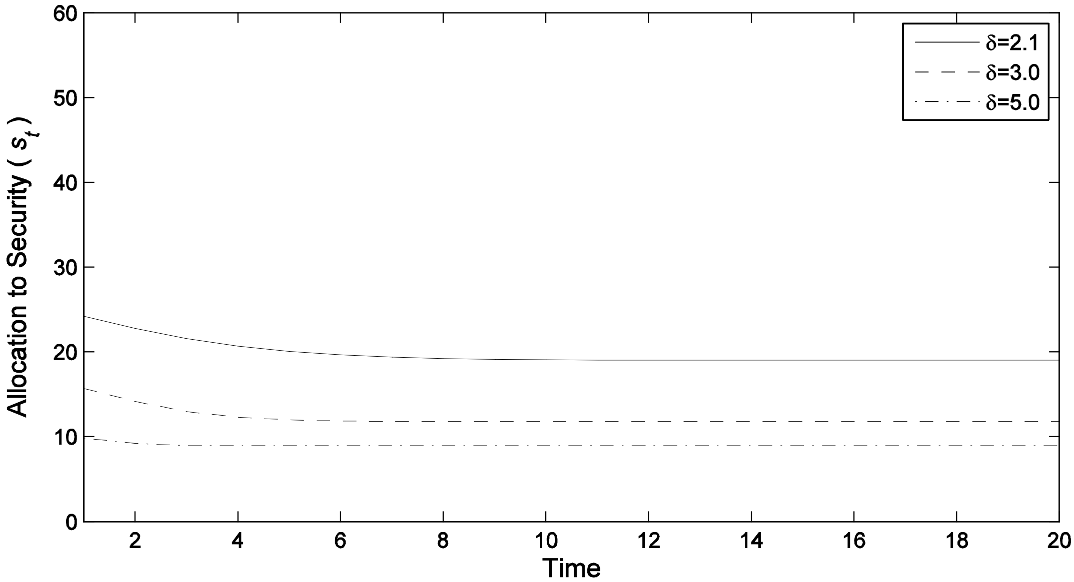

Figure 3f: Allocations to Security over Time- Different Values of Shape Parameter

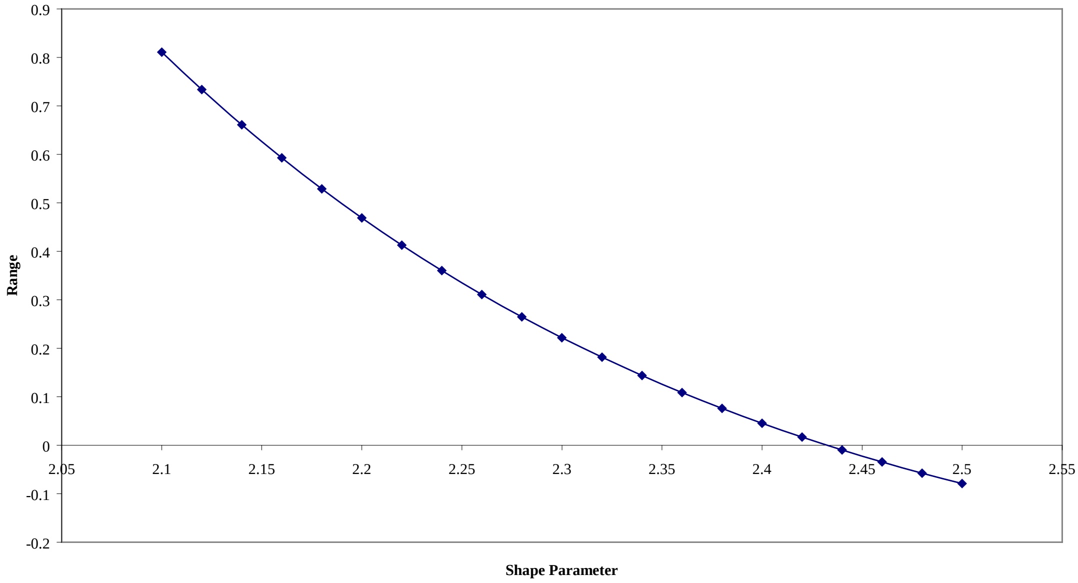

Figure 4a: Feasible Range of Insurance - Variation with Shape Parameter

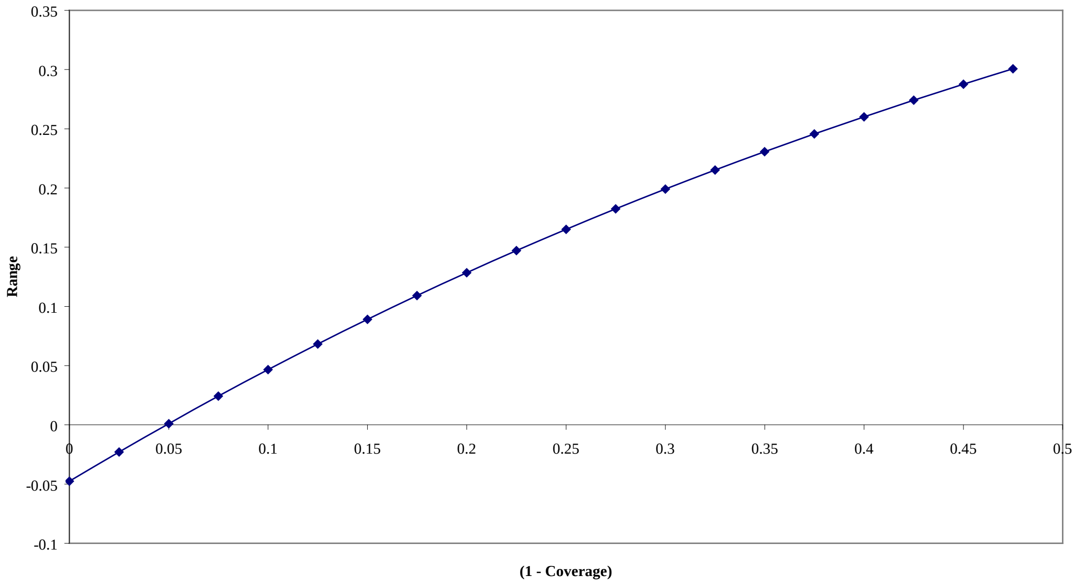

Figure 4b: Feasible Range of Insurance - Variation with Coverage

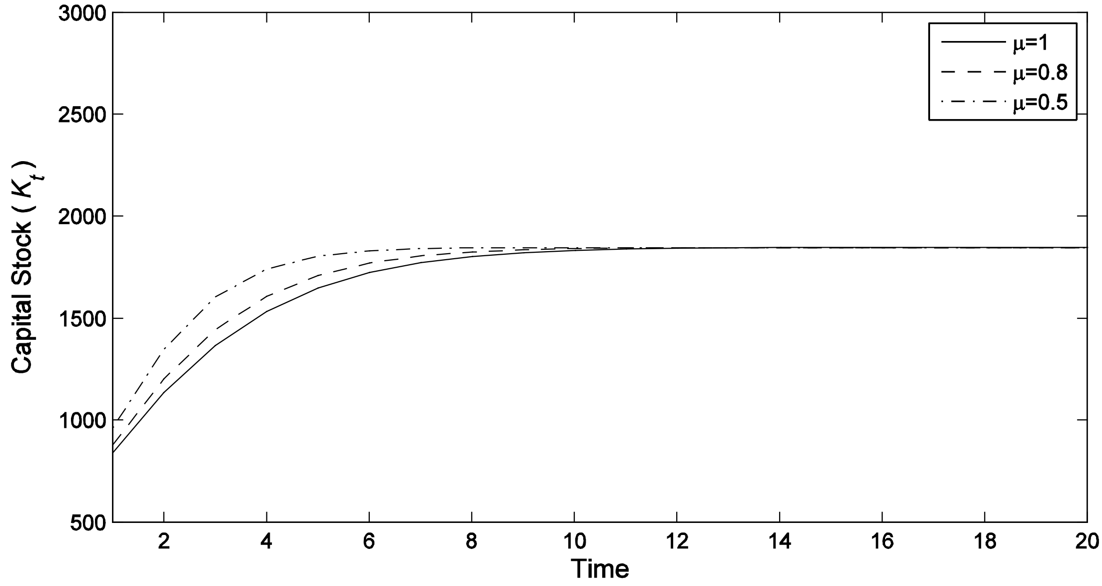

Figure 5a: Effect of Insurance Coverage on Capital Stock

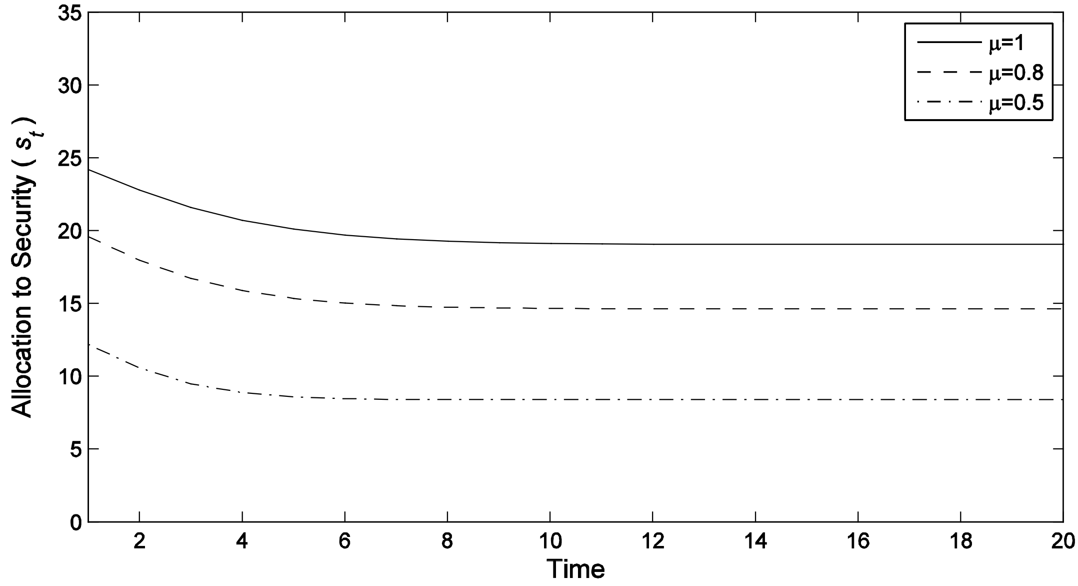

Figure 5b: Effect of Insurance Coverage on Allocation to Security